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ON DEGREE DOMINATION IN GRAPHS

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Abstract. A set S of vertices in a graph G is said to be a *dominating set* if every vertex $V - S$ is adjacent to some vertex in S. A *degree dominating function (DDF)* is a function $f : V(G) \to \{0, 1, 2, ..., \triangle(G) + 1\}$ having the property that every vertex v of S is assigned with $deg(v) + 1$ and all remaining vertices with zero. The weight of a degree dominating function f is defined by $w(f) = \sum_{v \in S} deg(v) + 1$. The *degree domination number*, denoted by γ_{deg}, is the minimum weight of all possible DDFs. In this paper, we introduced a new domination parameter called degree domination and studied the degree domination numbers of some graph types.

Keywords: domination number, dominating set, degree domination, degree dominating function.
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1 Introduction

Domination in graph theory is one of the most studied concepts that has attracted many researchers to study on it due to its many applications in many fields, such as in facility location problems where one attempts to minimize the distance that a person needs to travel to reach to the nearest facility.

A dominating set is a set S of vertices in a graph G if every vertex $V - S$ is adjacent to some vertex in S (Haynes et al., 1998). There are many types of domination depending on the structures of dominating sets. One of these types, the weighted domination number γ_w of (G, w) is the minimum weight $w(D) = \sum_{v \in D} w(v)$ of a set $D \subseteq V$ with $N[D] = V$, i.e. a dominating set of G (Dankelmann et al., 2004). The Roman domination number, denoted by γ_R, is the minimum weight of all possible RDFs which is defined as a function $f : V \to \{0, 1, 2\}$ satisfying the condition that every vertex v for which $f(v) = 0$ is adjacent to at least one vertex u for which $f(u) = 2$ (Hedetniemi et al., 2009). Also, (Cockayne et al., 1980), (Harary et al., 2000), (Kılıç et al., 2020), (Shalaan et al., 2020) are other studied types of dominating sets. The concept of the degree dominating function (DDF) will be defined in the next section.

One application of degree domination is that facilities for natural disasters and aid resources. Let the cities of a country be vertices and the roads between these cities be edges. We have to use domination while establishing facilities in case of a possible natural disaster in this country. The minimum number of facilities that should be established in the region is the domination number. We must store aid resources in these facilities so that we can send aid to cities in case of natural disaster. For this, we should use degree domination. While doing this, we will use the number of neighbors of the cities where the facilities are installed. Therefore, the sum of

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one more than the degree of each facility gives the degree domination number. Our aim here is to ensure that if we send aid to every city, the city where the facility is located will not be left without resources.

In this paper, first we introduced a new domination parameter called degree domination and studied the degree domination numbers of some standard graph types. Second, we find results on some known trees and finally, we studied some graph operation.

2 Degree Domination Number

In this section, we define a new domination parameter called degree domination number and denoted by γ_{deg}. We find results for γ_{deg} of some standard graphs such as path, cycle, star, complete, complete bipartite, wheel and we give proves of these results.

Definition 1. *The degree dominating function (DDF) is a function $f : V(G) \to \{0, 1, 2, ..., \triangle(G)+1\}$ having the property that every vertex v of the dominating set S is assigned with $deg(v) + 1$ and all remaining vertices with zero. The weight of a degree dominating function f is defined by $w(f) = \sum_{v \in S} deg(v) + 1$. The degree domination number, denoted by γ_{deg}, is the minimum weight of all possible DDFs.*

Example 1. Let examine the degree dominating set in given Figure 1.

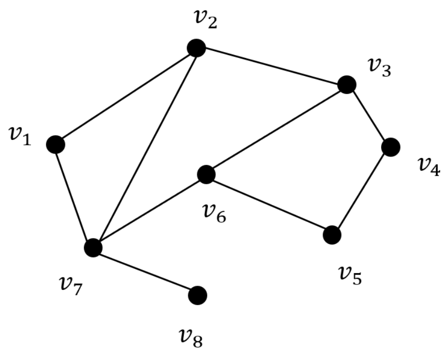

Figure 1: A graph G with eight vertices

In Figure 1, there are many dominating sets, but the set that gives the minimum weight should be chosen. It is seen that the set $S = \{v_4, v_7\}$ is the minimum dominating set. The maximum degree of the graph G is $\triangle(G) = 4$. By the definition of DDF, $f : V(G) \to \{0, 1, 2, 3, 4, 5\}$ and the DDF must consist of vertices $\{deg(v_4) + 1, deg(v_7) + 1\}$. Hence, the degree domination number is $\gamma_{deg}(G) = \sum_{v \in S} f(v) = (2 + 1) + (4 + 1) = 3 + 5 = 8$.

Theorem 1. *For $n \geqslant 3$,*

$$\gamma_{deg}(P_n) = \begin{cases} n, & \text{if } n \equiv 0, 2 (mod 3) \\ n+1, & \text{if } n \equiv 1 (mod 3) \end{cases}$$

Proof. Let $P_n = \{v_1, v_2, ..., v_n\}$ be a path of order n. It is known that the degree of all vertices except the pendant vertices is 2. Then, we define $f : V(P_n) \to \{0, 1, 2, 3\}$.

If $n \equiv 0(mod 3)$ then $f(v_{3i-1}) = 3$ for $1 \le i \le \frac{n}{3}$.

If $n \equiv 2(mod 3)$ then $f(v_{3i-1}) = 3$ for $1 \le i \le \frac{n-2}{3}$ and $f(v_n) = 2$.

If $n \equiv 1(mod 3)$ then $f(v_{3i-1}) = 3$ for $1 \le i \le \frac{n-1}{3}$ and $f(v_n) = 2$.

For all remaining vertices $f(v) = 0$. It is easy to generalize that f is DDF of P_n of weight $3.\frac{n}{3} = n$ if $n \equiv 0(mod3)$, $3.\frac{n-2}{3}+2 = n$ if $n \equiv 2(mod3)$ and $3.\frac{n-1}{3}+2 = n+1$ if $n \equiv 1(mod3)$. Thus,

$$\gamma_{deg}(P_n) = \begin{cases} n, & \text{if } n \equiv 0,2(mod3) \\ n+1, & \text{if } n \equiv 1(mod3) \end{cases}$$

□

Theorem 2. *For $n \geqslant 3$,*

$$\gamma_{deg}(C_n) = \begin{cases} n, & \textit{if } n \equiv 0(mod3) \\ n+1, & \textit{if } n \equiv 2(mod3) \\ n+2, & \textit{if } n \equiv 1(mod3) \end{cases}$$

Proof. Let $C_n = \{v_1, v_2, ..., v_n\}$ be a cycle of order n. It is known that the degree of all vertices is 2. Then, we define $f : V(C_n) \to \{0, 1, 2, 3\}$.

If $n \equiv 0(mod3)$ then $f(v_{3i-1}) = 3$ for $1 \leq i \leq \frac{n}{3}$.

If $n \equiv 2(mod3)$ then $f(v_{3i-1}) = 3$ for $1 \leq i \leq \frac{n-2}{3}$ and $f(v_n) = 3$.

If $n \equiv 1(mod3)$ then $f(v_{3i-1}) = 3$ for $1 \leq i \leq \frac{n-1}{3}$ and $f(v_n) = 3$.

For all remaining vertices $f(v) = 0$. It is easy to generalize that f is DDF of C_n of weight $3.\frac{n}{3} = n$ if $n \equiv 0(mod3)$, $3.\frac{n-2}{3} + 3 = n+1$ if $n \equiv 2(mod3)$ and $3.\frac{n-1}{3} + 3 = n+2$ if $n \equiv 1(mod3)$. Thus,

$$\gamma_{deg}(C_n) = \begin{cases} n, & \text{if } n \equiv 0(mod3) \\ n+1, & \text{if } n \equiv 2(mod3) \\ n+2, & \text{if } n \equiv 1(mod3) \end{cases}$$

□

Theorem 3. *For $n \geqslant 1$, $\gamma_{deg}(K_n) = n$.*

Proof. We know that K_n is a $(n-1)$-regular graph and $\gamma(K_n) = 1$. Let this vertex be v. By definition of DDF, $\gamma_{deg}(K_n) = deg(v) + 1 = n - 1 + 1 = n$.

□

Theorem 4. *For $n \geqslant 3$, $\gamma_{deg}(W_n) = \gamma_{deg}(S_n) = n + 1$.*

Proof. Let v be the central vertex. It is clear that $\gamma(W_n) = \gamma(S_n) = 1$ and $S = \{v\}$ is the dominating set of W_n and S_n. By definition of DDF, $\gamma_{deg}(W_n) = \gamma_{deg}(S_n) = deg(v)+1 = n+1$.

□

Theorem 5. *Let G be a complete bipartite graph where $m \geqslant n$, then $\gamma_{deg}(K_{m,n}) = m + n + 2$.*

Proof. Suppose that V_1 and V_2 are the bipartite sets of the graph G of order m and n, respectively. We know that $\gamma(K_{m,n}) = 2$ and $S = \{x, y | x \in V_1, y \in V_2\}$ is the dominating set of $K_{m,n}$. Then, $deg(x) = n$ and $deg(y) = m$. By definition of DDF, $\gamma_{deg}(K_{m,n}) = (deg(x) + 1) + (deg(y) + 1) = m + n + 2$.

□

Observation 1. *Let G be a null graph has n vertices, then $\gamma_{deg}(N_n) = n$.*

Theorem 6. *For any graph G with $n \geqslant 2$ vertices, $\gamma(G) \leq \gamma_{deg}(G)$.*

Proof. Suppose that S is a dominating set and D is the degree dominating set of G. Let $|S| = k$ with $k \geq 1$. It is clear that by the definition of DDF, D consists of $deg(v) + 1$ where $v \in S$. Thus, $|D| = \sum_{i=1}^{i=k} deg(v_i) + 1$. Therefore, $|S| \leq |D|$ and $\gamma(G) \leq \gamma_{deg}(G)$. □

Lemma 1. *For any connected graph G with $n \geqslant 2$ vertices, $\gamma_{deg}(G) \geq deg(v_i)$ where $1 \leq i \leq n$.*

Lemma 2. *Let G be r-regular graph, then $\gamma_{deg}(G) = (r + 1).\gamma_{deg}(G)$.*

Proof. Suppose that S is a dominating set and r is the degree of G. Let $|S| = k$ with $k \geq 1$. It is clear that the degree of all vertices of S is r. by the definition of DDF, $\gamma_{deg}(G) = \sum_{i=1}^{i=k} r + 1$. Therefore, $\gamma_{deg}(G) = (r + 1).k = (r + 1).|S| = (r + 1).\gamma_{deg}(G)$. □

3 Degree Domination Number of Some Known Trees

Trees are important not only for sake of their applications to many different fields, but also to graph theory itself. The very simplicity of trees makes it possible to investigate conjectures for graphs in general by first studying the situation for trees. Several ways of defining a tree are developed. The domination number of these several trees is being studied, for example (Jou et al., 2017) and (Demirpolat et al., 2021).

In this section, the degree domination number of some related networks namely binomial tree (B_n), complete k-ary tree (H_n^k) and E_n^k tree are studied.

Theorem 7. *Let B_n be a binomial tree, $\gamma_{deg}(B_n) = 2^n$.*

Proof. Let S be a dominating set of B_n. From the definition of binomial tree (Corrmen et al., 1990), the structure of B_n includes two B_{n-1}. With this logic, the recursive structure of the B_n obtained as $B_n = 2^i.(B_{n-i})$ for $1 \leq i \leq n - 2$. Let $n < 3$. It is easily seen that when $n = 0$ there is no leaf and there is only v isolated vertex. By definition of DDF, $\gamma_{deg}(B_0) = deg(v) + 1 = 0 + 1 = 1$. When $n = 1$, the number of leaves of B_1 is 1 and $\gamma_{deg}(B_1) = 2\gamma_{deg}(B_0) = 2$. When $n = 2$, $\gamma_{deg}(B_2) = 2\gamma_{deg}(B_1) = 4$. For $n \geq 3$, if we continue to do same logic, we obtain that degree dominating set of B_n is

$$\gamma_{deg}(B_n) = 2.\gamma_{deg}(B_{n-1}).$$

If we put this result in recursive formula, we have

$$\gamma_{deg}(B_n) = 2^i.(B_{n-i}), \text{ for } 1 \leq i \leq n - 1.$$

We prove this formula by induction on i for $n \geq 3$.

- Let $i = 0$, $\gamma_{deg}(B_0) = 1$.
- Let $i = 1$. Then, $\gamma_{deg}(B_1) = 2\gamma_{deg}(B_0) = 2$.
- Let $i = k$ and the result is true. We assume that $\gamma_{deg}(B_n) = 2^k.\gamma_{deg}(B_{n-k})$.

Now, we prove it for $i = k + 1$. Then we get

$$\gamma_{deg}(B_n) = 2^k.(2.\gamma_{deg}(B_{n-k-1})) = 2^{k+1}.\gamma_{deg}(B_{n-(k+1)}).$$

Hence the formula is true for $i = k + 1$ and we have $\gamma_{deg}(B_n) = 2^i.(B_{n-i})$, for $1 \leq i \leq n - 1$. We obtain the following formula by putting $i = n - 1$ in $\gamma_{deg}(B_n) = 2^i.(B_{n-i})$. Therefore,

$$\gamma_{deg}(B_n) = 2^i.(B_{n-i})$$

$$\gamma_{deg}(B_n) = 2^{n-1}.(B_{n-(n-1)})$$

$$\gamma_{deg}(B_n) = 2^{n-1}.(B_1)$$

$$\gamma_{deg}(B_n) = 2^{n-1}.2$$

$$\gamma_{deg}(B_n) = 2^n.$$

□

Theorem 8. *Let H_n^k be a complete k-ary tree, then*

$$\gamma_{deg}(H_n^k) = \begin{cases} (k+1)(\sum_{i=0}^{\lceil \frac{n-3}{3} \rceil} k^{n-1-3i}) + k.1, & \text{if } n \equiv 0(mod3) \\ (k+1)(\sum_{i=0}^{\lfloor \frac{n-3}{3} \rfloor} k^{n-1-3i}) + k.1, & \text{if } n \equiv 1(mod3) \\ (k+1)(\sum_{i=0}^{\lceil \frac{n-3}{3} \rceil} k^{n-1-3i}), & \text{if } n \equiv 2(mod3) \end{cases}$$

.

Proof. Let S be a dominating set and V' be the degree dominating set. By the definition of complete k-ary tree (Corrmen et al., 1990), H_n^k has k children for every vertices except leaves. Let v be the root vertex at zeroth level. To dominate k^n vertices at nth level, all k^{n-1} vertices at $(n-1)$st level must be added to S. Thus all vertices at nth level and $(n-2)$th level are dominated. Similarly, all vertices at $(n-3)$th level and $(n-5)$th level are dominated by taking of all vertices at $(n-4)$st level to S. Here we have three cases depending on n.

If $n \equiv 0(mod3)$ then all vertices at $n-1, n-4, ..., n-1-3\lceil \frac{n-3}{3} \rceil$ levels are added to S. Thus $|S| = \sum_{i=0}^{\lceil \frac{n-3}{3} \rceil} k^{n-1-3i}$. The vertex v is not dominated and we must add this vertex to S. Thus $S \cup \{v\}$ is the dominating set of H_n^k. All vertices in S except v have degree $k+1$ and that of v has degree k. By the definition of DDF, $|V'| = (k+1)(\sum_{i=0}^{\lceil \frac{n-3}{3} \rceil} k^{n-1-3i}) + k.1$.

If $n \equiv 1(mod3)$ then similarly all vertices at $n-1, n-4, ..., n-1-3\lfloor \frac{n-3}{3} \rfloor$ levels are added to S. Thus $|S| = \sum_{i=0}^{\lfloor \frac{n-3}{3} \rfloor} k^{n-1-3i}$. The vertex v is not dominated and we must add this vertex to S. Hence, $S \cup \{v\}$ is the dominating set of H_n^k. All vertices in S except v have degree $k+1$ and that of v has degree k. By the definition of DDF, $|V'| = (k+1)(\sum_{i=0}^{\lfloor \frac{n-3}{3} \rfloor} k^{n-1-3i}) + k.1$.

If $n \equiv 2(mod3)$ then it seen that all vertices of H_n^k are dominated by S and $|S| = \sum_{i=0}^{\lceil \frac{n-3}{3} \rceil} k^{n-1-3i}$. Thus, by the definition of DDF, $|V'| = (k+1)(\sum_{i=0}^{\lceil \frac{n-3}{3} \rceil} k^{n-1-3i})$.

By cases, the proof is completed.

□

Theorem 9. *Let E_n^k be a tree then, $\gamma_{deg}(E_n^k) = k.n + 2$.*

Proof. The graph E_n^k is a tree which has k legs and each leg has n vertices (Corrmen et al., 1990). Let $x, y, v_{i,j}$ be the vertices of E_n^k, where $i \in \{1, 2, ..., k\}$, $j \in \{1, 2, ..., n\}$. y is the vertex having maximum degree, which is $deg(y) = k+1$ and x is the adjacent vertex to y of degree 1. Thus, we define $f : V(G) \to \{0, 1, 2, ..., (k+1)+1\}$. There are three cases to examine.

If $n \equiv 0(mod3)$ then $f(v_{i,3j-1}) = 3$ for $1 \le i \le k$, $1 \le j \le \dfrac{n}{3}$ and $f(x) = 2$.

If $n \equiv 2(mod3)$ then $f(v_{i,3j-1}) = 3$ and $f(v_{i,n}) = 2$ for $1 \le i \le k$, $1 \le j \le \dfrac{n-2}{3}$ and $f(x) = 2$.

If $n \equiv 1(mod3)$ then $f(v_{i,3j}) = 3$ for $1 \le i \le k$, $1 \le j \le \dfrac{n-1}{3}$ and $f(y) = k+2$.

For all remaining vertices $f(v_{i,j}) = 0$. It easy to see that E_n^k includes k paths. Then, we generalize f of P_n of weight $k.(3.\dfrac{n}{3}) + 2 = k.n + 2$ if $n \equiv 0(mod3)$, $k.(3.\dfrac{n-2}{3} + 2) + 2 = k.n + 2$ if $n \equiv 2(mod3)$ and $k.(3.\dfrac{n-1}{3}) + k + 2 = k.n + 2$ if $n \equiv 1(mod3)$. Thus, $\gamma_{deg}(E_n^k) = k.n + 2$.

□

4 Degree Domination Number of Some Graph Operation

In this section, we study the degree domination number of some graph operation such as $P_n+\overline{K_m}$, cartesian product (Imrich et al., 2008) such $C_n \times P_2$ and corona (Buckley et al., 1990) of $C_n \odot \overline{K_m}$, $P_2 \odot P_n$ and $P_n \odot K_m$.

Theorem 10. *Let $G \equiv C_n \times P_2$ where $n \geq 3$, then $\gamma_{deg}(G) = 4(\lceil \frac{n-3}{4} \rceil + \lceil \frac{n-5}{4} \rceil)$.*

Proof. Let $V = \{v_1, v_2, ..., v_n\}$ be a first cycle of C_{1n}, $U = \{u_1, u_2, ..., u_n\}$ be a second cycle of C_{2n} and S be a dominating set of G. The degree of all vertices of the graph is $deg(v) = 3$. By the definition of DDF, $deg(v) + 1 = 3 + 1 = 4$. Therefore, the degree dominating set for C_{1n} is determined as $f(v_{1+4i}) = 4$ for $0 \leq i \leq \lceil \frac{n-3}{4} \rceil$, while for C_{2n} is determined as $f(u_{3+4j}) = 4$ for $0 \leq j \leq \lceil \frac{n-5}{4} \rceil$. For all remaining vertices $f(v) = 0$. Thus, $\gamma_{deg}(G) = 4(\lceil \frac{n-3}{4} \rceil + \lceil \frac{n-5}{4} \rceil)$. □

Theorem 11. *Let $G \equiv P_n + \overline{K_m}$ where $m \leq n$, then $\gamma_{deg}(G) = n + m + 2$.*

Proof. Let $P_n = \{v_1, v_2, ..., v_n\}$ be a path of order n and $\overline{K_m} = \{u_1, u_2, ..., u_m\}$ be a set of vertices of order m. It easy to see that $S = \{v_1, u_1\}$ is the dominating set of G because v_1 dominates all vertices formed by $\overline{K_m}$ and u_1 dominates all vertices of P_n. We know that $deg(v_1) = m$ and $deg(u_1) = n$. By the definition of DDF, $\gamma_{deg}(G) = n + m + 2$. □

Theorem 12. *Let $G \equiv C_n \odot \overline{K_m}$, then*

$$\gamma_{deg}(G) = \begin{cases} n.(2m), & \text{if } m < 3 \\ n.(m+2), & \text{if } m \geq 3 \end{cases}$$

Proof. Let $m < 3$ and $U = \{u_1, u_2, ..., u_m\}$ be a set of pendant vertices of $\overline{K_m}$. It is easy to verify that $S_1 = \{u_1, u_2, ..., u_m\}$ is the dominating set to dominate all vertices of C_n. The degree of each vertex in S_1 is 1. By the definition of DDF, $\gamma_{deg}(G) = n.(2m)$.

Let $m \geq 3$ and $V = \{v_1, v_2, ..., v_n\}$ be a cycle of C_n. It is to verify that $S_2 = \{v_1, v_2, ..., v_n\}$ is the dominating set to dominate the pendant vertices formed by $\overline{K_m}$. The degree of each vertex in S_2 is $(m + 2)$. By the definition of DDF, $\gamma_{deg}(G) = n.(m + 2)$. □

Theorem 13. *Let $G \equiv P_2 \odot P_n$ with $n \geq 4$, then $\gamma_{deg}(G) = 2n + 4$.*

Proof. $P_2 \odot P_n$ is formed by taking one copy of P_2 where the vertices are $\{u_1, u_2\}$ and 2 copies of P_2 where the vertices sets are $V_1 = \{v_{11}, v_{12}, ..., v_{1n}\}$ and $V_2 = \{v_{21}, v_{22}, ..., v_{2n}\}$. It easy to see that $S = \{u_1, u_2\}$ is the dominating set of G, since u_1 dominates V_1 and u_2 dominates V_2. By the definition of DDF, $\gamma_{deg}(G) = (deg(u_1)+1)+(deg(u_2)+1) = (n+1+1)+(n+1+1) = 2n+4$. □

Theorem 14. *Let $G \equiv P_n \odot K_m$ with $n, m \geq 2$, then $\gamma_{deg}(G) = n(m + 1)$.*

Proof. $P_n \odot K_n$ is formed by taking one copy of P_n where the vertices are $\{v_1, v_2, ..., v_n\}$ and n copies of K_m where the vertices sets are $U_i = \{u_{i1}, u_{i2}, ..., u_{im}\}$ with $1 \leq i \leq n$. It easy to see that $S = \{u_{11}, u_{21}, ..., u_{n1}\}$ is the dominating set of G and the degree of all these vertices is m. By the definition of DDF, $\gamma_{deg}(G) = (deg(u_{11}) + 1) + (deg(u_{21}) + 1) + ... + (deg(u_{n1}) + 1) = (m + 1) + (m + 1) + ... + (m + 1) = n(m + 1)$.

□

5 Conclusion

In graph theory, domination is one of the most important parameters in the stability and vulnerability analysis of communication networks modeled by graphs. There are various types of domination depending on structure and properties of dominating sets.

In this paper, we introduced a new domination parameter called degree domination number. We obtain results of this number on some certain graph classes such as P_n, C_n, S_n, W_n, K_n, $K_{m,n}$ and some known trees such as B_n, H_n^k, E_n^k. Also, some graph operations such as $C_n \times P_2$, $C_n \odot \overline{K_m}$, $P_2 \odot P_n$ and $P_n \odot K_m$ are determined.

The study on degree domination offers much scope for further research. We will extend this study for future work under some other graph operations, on edge set of graphs and also on different dominating sets such as total domination.

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